

## ENHANCING THE CONTROLLABILITY OF A COMPOSITE DIELECTRIC

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*The possibility of increasing the control coefficient of a nonlinear dielectric is studied. Composite structures are designed for which the control coefficient of the composite is considerably (5–20 times) higher than the control coefficients of its components are developed.*

**Key words:** *ferroelectric–dielectric composite, control coefficient, control gain coefficient, dielectric loss.*

**1. Control Coefficient of a Nonlinear Composite Dielectric.** In the electrotechnical industry, use is made of materials with a nonlinear dependence of the dielectric constant on the intensity of the applied electric field [1–4]. A characteristic of nonlinearity is the quantity  $T$ , which is called the relative control of a nonlinear dielectric [1, 2]:

$$T = (\varepsilon(E) - \varepsilon(0))/\varepsilon(0) \quad (1)$$

$[\varepsilon(E)$  is a dielectric constant that corresponds to the electric-field intensity  $E = |\nabla\varphi|$ ;  $\varphi(\mathbf{x})$  is the electric-field potential;  $\mathbf{x} = (x, y)$  or  $\mathbf{x} = (x, y, z)$  depending on the dimension of the problem]. This quantity characterizes the change in the dielectric properties of a material under the action of a constant electric field, which is used in the design of control devices [1, 3, 4].

The function  $\varepsilon(E)$  is usually adopted in the form

$$\varepsilon(E) = \varepsilon_0 + \mu E^2. \quad (2)$$

Here  $\varepsilon_0$  is the dielectric constant at zero (more precisely, at  $E \rightarrow 0$ ). Although experimental curves for large values of  $E$  do not coincide with the curve described by dependence (2), the latter can be used for weak fields as an approximation [in many studies, (2) is used for arbitrary field magnitudes]. If the function  $\varepsilon(E)$  is chosen in the form of (2), formula (1) becomes

$$T = \mu E^2 / \varepsilon_0.$$

Materials with high control coefficients (for the currently used materials, a control coefficient of 10–20% is considered high) are characterized by a large dielectric constant ( $\varepsilon_0 = 500$ –1000) and fairly large losses [ $\tan \delta = (50$ –100) · 10<sup>−4</sup>]. To reduce the dielectric constant and dielectric loss, it has been proposed [1, 3, 4] to use composite materials. However, the effectiveness of using these materials has not been determined because of the absence of information on the possible values of the average control of composites.

Tagantsev et al. give the following formulas for calculating the dielectric constant  $\varepsilon_{\text{mix}}$  and control  $T_{\text{mix}}$  for a medium filled with spheres:

$$\varepsilon_{\text{mix}} = \varepsilon(1 - 1.5q), \quad T_{\text{mix}} = n_r(1 + 0.2q) \quad (3)$$

( $q$  is the volume fraction of spheres). Formulas (3) are based on the solution of the electrostatic problem for one sphere [5] and were obtained under the assumption of smallness of  $q$ . According to (3), the possible increase in the

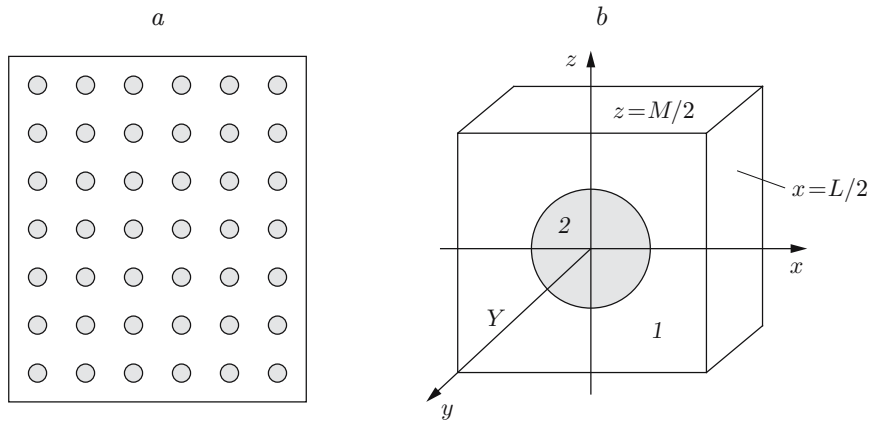


Fig. 1. Model of a composite material of periodic structure (a) and the periodicity cell  $Y$  (b): 1) material No. 1; 2) material No. 2.

control does not exceed 3–5%. One should take into account that the formulas obtained with the assumption  $q \ll 1$  are usually valid for  $q \leq 0.1$ – $0.2$ . Experimental data [1] indicate that the control depends weakly on the volume of dielectric particles at  $q = 0.1$ – $0.2$ , which agrees with the predicted [6] weaker decrease in the lower-bound estimate for  $T_{\text{mix}}$  compared with the decrease in  $\varepsilon_{\text{mix}}$ .

Thus, at present, there is no consensus on the possible values of the control of a composite (the parameter values can change in the required direction by a few percent or even a few tens of percent) and on the effect of the microstructure of composite on this parameter.

The numerical calculations of the characteristics of composites for various microstructures performed in the present work allow one to estimate the efficiency of existing design solutions and to develop methods for improving material characteristics.

**2. Effective Dielectric Constant of a Nonlinear Composite of Periodic Structure (Case of Small Fields).** Methods for calculating the control coefficient of a composite from the characteristics of its components have been considered in many papers (see, for example, [1, 5, 7, 8]). In [6], the control coefficient of a composite was calculated using the mathematically rigorous averaging theory [9] for the two-dimensional problem. In the present work, we consider the three-dimensional problem and need to obtain the corresponding formulas for the three-dimensional case. Below, it is shown how the calculations of [6] should be modified for the case of the three-dimensional problem.

Let us consider a composite of periodic structure (Fig. 1) with a periodicity cell  $Y = [-L/2, L/2]^2 \times [-M/2, M/2]$ . Since the material properties of the composite material depend on the spatial variable  $\mathbf{x}$ , relation (2) is written as

$$\varepsilon(\mathbf{x}, |\nabla\varphi|) = \varepsilon_0(\mathbf{x}) + \mu(\mathbf{x})|\nabla\varphi|^2.$$

The induction vector  $\mathbf{I}$  is related to  $\nabla\varphi$  by the formula  $\mathbf{I} = \varepsilon(\mathbf{x}, |\nabla\varphi|)\nabla\varphi$ . The electric-field potential is determined by solving the equation

$$-er[\varepsilon(\mathbf{x}, |\nabla\varphi|)\nabla\varphi] = 0 \tag{4}$$

provided that the function  $\varphi(\mathbf{x}) - Ez$  is periodic in  $Y$ .

Let the coefficient  $\varepsilon(\mathbf{x}, \nabla\varphi)$  be given as

$$\varepsilon(\mathbf{x}, |\nabla\varphi|) = \varepsilon_0(\mathbf{x}) + \lambda\mu(\mathbf{x})|\nabla\varphi|^2,$$

where  $\lambda$  is a small parameter. This parameter can characterize not only the smallness of physical nonlinearity but also the smallness of the field in the case of considerable physical nonlinearity [6].

The averaged dielectric constant of the composite is calculated by the formula [9]

$$D = \frac{1}{E|Y|} \int_Y \varepsilon(\mathbf{x}, |\nabla\varphi|) |\nabla\varphi|^2 d\mathbf{x},$$

where  $|Y|$  is the volume of the periodicity cell  $Y$  of the composite.

Let the periodicity cell be symmetric about the coordinate planes. Then, the periodicity conditions can be replaced by mixed boundary conditions, i.e., it can be assumed that the horizontal faces of the periodicity cell are acted upon by the stresses  $\pm EM$  (these values correspond to the average stress  $E$ ), and on the vertical faces, the normal component of the field intensity is equal to zero. The symmetry conditions and the equivalence of the periodicity cells lead to the boundary conditions

$$\varphi(x, y, \pm M/2) = \pm EM/2; \quad (5)$$

$$\frac{\partial\varphi}{\partial\mathbf{n}} \left( \pm \frac{L}{2}, y, z \right) = \frac{\partial\varphi}{\partial\mathbf{n}} \left( x, \pm \frac{L}{2}, z \right) = 0. \quad (6)$$

The further calculations can be performed similarly to the calculations for the two-dimensional case [6]. We consider the case of weak nonlinearity, where  $\lambda \ll 1$ . Following [10], we seek a solution of the cell problem (4)–(6) in the form of an expansion in the small parameter  $\lambda$ :

$$\varphi(\mathbf{x}) = \varphi_0(\mathbf{x}) + \lambda\varphi_1(\mathbf{x}) + \dots \quad (7)$$

Substituting (7) into (4)–(6) and performing transformations similar to those in [6], we obtain

$$D = F/E = A + \lambda B, \quad (8)$$

where the quantities  $A$  and  $B$  are defined by the formulas

$$A = \frac{1}{|Y|} \int_Y \varepsilon_0(\mathbf{x}) |\nabla N(\mathbf{x})|^2 d\mathbf{x}; \quad (9)$$

$$B = \frac{1}{|Y|} \int_Y \mu(\mathbf{x}) |\nabla N(\mathbf{x})|^4 d\mathbf{x}, \quad (10)$$

$N(\mathbf{x})$  is a solution of the linear cell problem

$$-er [\varepsilon_0(\mathbf{x}) \nabla N] = 0, \quad (11)$$

$$N \left( x, y, \pm \frac{M}{2} \right) = \pm \frac{1}{2}, \quad \frac{\partial N}{\partial\mathbf{n}} \left( \pm \frac{L}{2}, y, z \right) = \frac{\partial N}{\partial\mathbf{n}} \left( x, \pm \frac{L}{2}, z \right) = 0.$$

In averaging theory [9], formula (9) is known as the formula for the averaged constant of a linear composite, and formula (11) as the cell problem for a linear composite.

**3. Control Gain Coefficient of a Composite.** From (8) it follows that the averaged dielectric constant of a composite  $D(E)$ , which relates the averaged induction vector  $\mathbf{I}$  and the averaged electric-field intensity  $\nabla\varphi$ , has the form

$$D(E) = A + BE^2, \quad (12)$$

where  $A$  and  $B$  are defined by formulas (9) and (10). The fact that Eq. (12) includes a terms containing the field intensity to a fourth power agrees with the results of [7, 8]. We note that in [7, 8] this problem was considered at a physical level of rigorosity. In the present work, we use a mathematically rigorous method of averaging [9]. In this case, formula (12) is valid only for weak nonlinearity or weak fields. There is no reason to extend formula (12) to the general case.

The averaged control coefficient corresponding to formula (12) is equal to

$$K_{\text{mix}} = BE^2/A.$$

According to formulas (9) and (10), the control of the composite is determined by the quantity

$$t = \frac{B}{A} = \int_Y \mu(\mathbf{x}) |\nabla N(\mathbf{x})|^4 d\mathbf{x} / \int_Y \varepsilon_0(\mathbf{x}) |\nabla N(\mathbf{x})|^2 d\mathbf{x}.$$

If only the matrix material is nonlinear, then

$$t = \frac{B}{A} = \mu_M \int_M |\nabla N(\mathbf{x})|^4 d\mathbf{x} / \int_Y \varepsilon_0(\mathbf{x}) |\nabla N(\mathbf{x})|^2 d\mathbf{x}.$$

Here  $M$  is the region occupied by the matrix; the subscript  $M$  denotes the characteristics of the matrix material. The quantity

$$K = t/t_M \quad (13)$$

is called the control gain coefficient [11] and characterizes the increase (decrease) in the control coefficient of the composite in comparison with the control coefficient of the matrix material ( $t_M = \mu_M/\varepsilon_M$  is the control of the matrix material). Definition (13) can be extended to the case of composites containing several nonlinear components.

From formulas (9) and (10), it follows that, in the calculation of the averaged characteristics, the main difficulty is to calculate the function  $N(\mathbf{x})$ . This problem can be solved by numerical methods. Below, we give the results of solution of problem (11) and calculate the quantities  $A$  and  $B$  in (9) and (10) using the ANSYS code.

**4. Numerical Analysis of the Problem and Design of Composites.** The typical characteristics of the materials used as components of composites are given below. Ferroelectric is used as a nonlinear material. For such a material, the dielectric constant is  $\varepsilon = 500\text{--}2000$ , the relative control is  $t = 10\text{--}20\%$ , and the dielectric loss tangent is  $\tan \delta = (50\text{--}100) \cdot 10^{-4}$ . The linear material (dielectric) has the following characteristics:  $\varepsilon = 1\text{--}10$ ,  $t = 0$ , and  $\tan \delta = (1\text{--}5) \cdot 10^{-4}$  [1].

We note that changes in the properties of a composite are determined, as a rule, by relative rather than absolute values of the characteristics of the constituent materials; therefore, the calculations use relative values.

**4.1. Spherical Inclusions.** We consider a composite which consists of a matrix with a periodic system of spherical inclusions. Similar composites of irregular structure were considered in [1, 2, 4, 7, 8].

In the numerical calculation, it was assumed that  $\varepsilon_1 = 100$  and  $\varepsilon_2 = 1$  (the subscript 1 corresponds to the nonlinear matrix material, and the subscript 2 to the linear material of the inclusion). The controls are  $t_1 = 0.01$  and  $t_2 = 0$ . The size of the periodicity cell is equal to  $1 \times 1$ , and the radius of the inclusion is 0.45. The volume fraction of the dielectric is 0.63.

The numerically obtained values of integrals (9) and (10) are equal, respectively, to

$$\int_Y \varepsilon_0(\mathbf{x}) |\nabla N(\mathbf{x})|^2 d\mathbf{x} = 52, \quad \int_Y \mu(\mathbf{x}) |\nabla N(\mathbf{x})|^4 d\mathbf{x} = 1.7.$$

The first integral is equal to the averaged dielectric permeability of the composite (since, in the case considered,  $|Y| = 1$ ). For the composite,  $t = 1.7/52 \approx 0.033$ . The control gain coefficient of a composite is  $K \approx 0.033/0.01 = 3.3$  (330%).

It is of interest to determine the dielectric loss factor in the composite. The problem of calculating the loss in nonlinear composites has not been solved completely, and, according to the formula used currently for linear composites, the imaginary part of the dielectric constant of a composite  $\varepsilon''$ , which determines the loss in the composite, is found from the equality

$$\varepsilon'' = \varepsilon_1'' \int_M |\nabla N(\mathbf{x})|^2 d\mathbf{x} + \varepsilon_2'' \int_I |\nabla N(\mathbf{x})|^2 d\mathbf{x},$$

where  $\varepsilon_1''$  and  $\varepsilon_2''$  are the imaginary parts of the dielectric constant of the components of the composite,  $\varepsilon''$  is the imaginary part of the dielectric constant of the composite, and  $M$  and  $I$  are the regions occupied by the matrix and inclusion, respectively. In the case considered,  $\varepsilon'' = 0.62\varepsilon_1'' + 0.64\varepsilon_2''$  (0.62 and 0.64 are the values of the corresponding integrals). The dielectric loss tangent is  $\tan \delta = \varepsilon''/A$ . The dielectric loss tangent ( $\tan \delta = \varepsilon''/\varepsilon$ ) for ferroelectrics is equal to  $\tan \delta_1 = 10^{-2}$ , and for dielectrics, it is  $\tan \delta_2 = 10^{-4}$ . Accordingly,  $\varepsilon_1'' = 1$ ,  $\varepsilon_2'' = 10^{-4}$ , and the imaginary part of the dielectric constant of the composite is  $\varepsilon'' \approx 0.62$ .

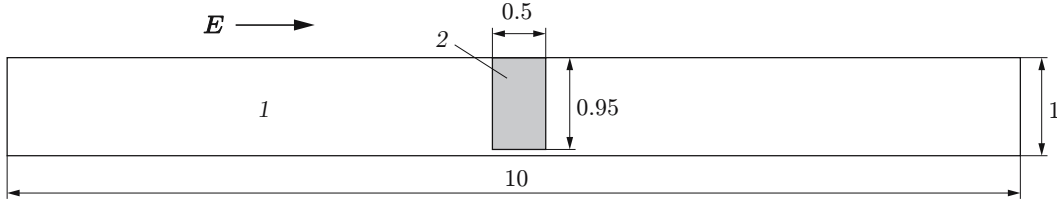


Fig. 2. Diagram of a ferroelectric cell (1) containing a dielectric insert (2).

The numerical calculations predict that, as the dielectric constant is decreased by approximately 40%, the control of the composite increases by a factor of more than three and the imaginary part of the dielectric constant of the composite  $\varepsilon''$  decreases by approximately 40% compared to the imaginary part of the ferroelectric  $\varepsilon_1'' = 1$ . The solution is easy to use in developing composite production processes.

4.2. *Enhancing the control by means of slots.* We consider designs of composites that considerably increase the control and determine what can be responsible for this increase.

The numerical calculation was performed for  $\varepsilon_1 = 1000$ ,  $\varepsilon_2 = 1$ ,  $t_1 = 0.001$ , and  $t_2 = 0$  (the subscript 1 corresponds to the nonlinear matrix material, and the subscript 2 to the linear insert material). The size of the periodicity cell is  $1 \times 10$ , the size of the insert is  $0.50 \times 0.95$ , and the slot width is 0.05 (Fig. 2). The volume fraction of the dielectric is 0.0475. Periodic repetition of the cell results in a structure similar to a membrane with perforations through which an electric field flows. The voltage drop across the membrane produces a high gradient in the slots occupied by the nonlinear material. Raising the gradient to the fourth power leads to large values of the integrand in (10) for the slot and yields a large value of the integral, despite the small area of the slots.

The numerically obtained values of the integrals are equal to

$$\int_Y \varepsilon_0(\mathbf{x}) |\nabla N(\mathbf{x})|^2 d\mathbf{x} = 4460, \quad \int_Y \mu(\mathbf{x}) |\nabla N(\mathbf{x})|^4 d\mathbf{x} = 159.$$

To obtain the average dielectric constant, it is necessary to divide the first integral by  $|Y| = 10$ . As a result, we obtain  $A = 446$ . For the composite,  $t = 159/4460 \approx 0.036$ . The control gain coefficient of the composite is  $K \approx 0.036/0.001 \approx 36$  (3600%). The dielectric loss in the composite is calculated by the formula

$$\varepsilon'' = \varepsilon_1'' \int_M |\nabla N(\mathbf{x})|^2 d\mathbf{x} + \varepsilon_2'' \int_I |\nabla N(\mathbf{x})|^2 d\mathbf{x} = 0.44\varepsilon_1'' + 6.5\varepsilon_2''.$$

For  $\tan \delta_1 = 10^{-2}$  and  $\tan \delta_2 = 10^{-4}$ , we have  $\varepsilon'' \approx 0.044$ . Accordingly, for the composite,  $\tan \delta = 0.044/44.6 \approx 10^{-2}$ .

The solution depends greatly on the ratio of the slot length to width. To choose these quantities, it was necessary to solve a number of problems (several iterations of the shape and size design problem). In the case of an inappropriate choice of the slot size, a considerable increase in the control is not observed.

4.3. *Enhancing the control by means of holes.* A three-dimensional analog of a slot is a hole. In the numerical calculation, the following values were used:  $\varepsilon_1 = 1000$ ,  $\varepsilon_2 = 1$ , controls  $t_1 = 0.001$ , and  $t_2 = 0$  (the subscript 1 corresponds to the nonlinear matrix material, and the subscript 2 to the linear insert material). A periodicity cell containing a hole is shown schematically in Fig. 3. Regions 1 and 3 are occupied by a ferroelectric, and region 2 is occupied by a dielectric. The insert can be understood as a perforated layer of a dielectric with hole 3 filled with a ferroelectric.

Periodic recurrence of the cell results in a perforated layer (Fig. 3) with an electric field passing through the holes. The voltage drop across the perforated layer produces a high gradient in region 3 (in the hole) occupied by the nonlinear material. Raising the gradient to the fourth power leads to large values of the integrand in (10) for region 3 and yields a large value of the integral, despite the small volume of the region 3 of holes. In the design considered, the volume fraction of the dielectric is about 0.09.

The numerically obtained values of the integrals are equal to

$$\int_Y \varepsilon_0(\mathbf{x}) |\nabla N(\mathbf{x})|^2 d\mathbf{x} = 12, \quad \int_Y \mu(\mathbf{x}) |\nabla N(\mathbf{x})|^4 d\mathbf{x} = 0.28.$$

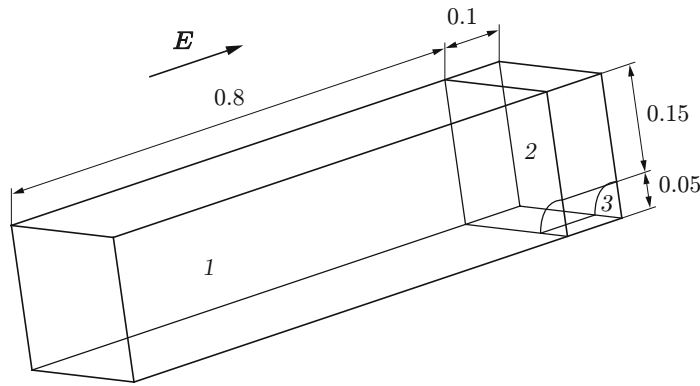


Fig. 3. Periodicity cell of composite: 1) ferroelectric; 2) dielectric insert; 3) hole occupied by a ferroelectric.

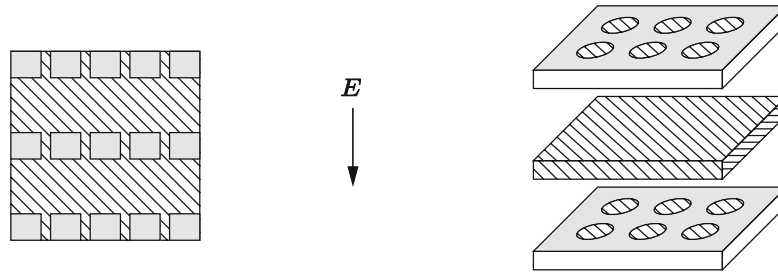


Fig. 4. Diagrams of two- and three-dimensional layered composites with a perforated layer.

To obtain the averaged dielectric constant, it is necessary to divide the first integral by  $|Y| = 0.036$ . For the composite,  $t = 0.28/12 \approx 0.023$ . The control gain coefficient of the composite is  $K \approx 0.023/0.001 \approx 23$  (2300%).

The last two designs correspond to layered composites, in which one layer is perforated (Fig. 4). Layered composites have averaged anisotropic characteristics. The question arises of whether it is possible to design an isotropic composite that enhances the control of the components. One such designs is a composite with spherical inclusions (see Fig. 1). However, its control gain coefficient is rather insignificant:  $K = 330\%$ . Perforated layers can be used to design cellular structures such as walls with holes (Fig. 5) with a large control gain coefficient. If a field is applied in a direction parallel to the coordinate axis, it is little affected by the walls parallel to the direction of the field, and the wall perpendicular to the direction of the field acts as a perforated layer.

4.4. *Composite Based on Nonlinear Materials.* The dielectric constant of ferroelectrics varies in the range  $\varepsilon = 500\text{--}2000$  [1], which allows one to design weak-contrast composites based on ferroelectrics, in which electric-field concentration occurs, resulting in a control gain. The numerical calculation was performed for  $\varepsilon_1 = 1$  and  $\varepsilon_2 = 5$  (the subscript 1 corresponds to the matrix material, and the subscript 2 to the inclusion material; both the inclusion and the matrix are ferroelectrics). The controls of the inclusion and matrix materials are set equal to  $t_2 = 1$  and  $t_1 = 0.2$  (for both materials,  $\mu = 1$ ). In the case considered, all values of the material constants are relative. The cell size is  $2 \times 2$ , and the inclusion radius is 0.9. The volume fraction of the inclusion is 0.63.

The numerically obtained values of the integrals are equal to

$$A = \frac{1}{|Y|} \int_Y \varepsilon_0(\mathbf{x}) |\nabla N(\mathbf{x})|^2 d\mathbf{x} = 1.9, \quad B = \frac{1}{|Y|} \int_Y \mu(\mathbf{x}) |\nabla N(\mathbf{x})|^4 d\mathbf{x} = 5.1.$$

For the composite  $t = 5.1/1.9 \approx 2.6$ . The control gain coefficient of the composite (with respect to the maximum value equal to unity)  $K = 2.6/1 = 2.6$  (260%).

The factor responsible for the enhancement of the control is the occurrence of an electric-field concentrator in the form of an energetic neck [11] (Fig. 6). However, the electric-field concentration in the neck does not reach values comparable to those in a slot or a hole (see Secs. 4.2 and 4.3).

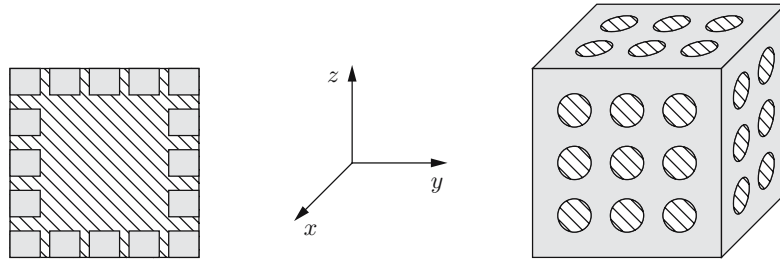


Fig. 5. Microstructures of the type of a wall with holes in two- and three-dimensional composites (the dashed region is the region occupied by a nonlinear material).

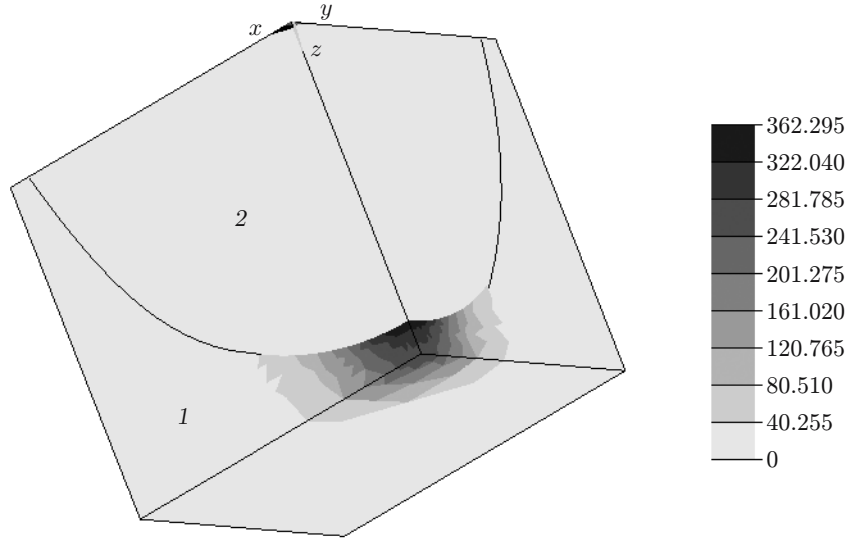


Fig. 6. Distribution of the quantity  $|\nabla N(\mathbf{x})|^4$  in a cell containing a spherical inclusion: 1) matrix (ferroelectric); 2) inclusion (ferroelectric).

**5. Dielectric Loss Tangent of a Composite Material.** As noted above, calculating losses in a nonlinear composite material is an interesting and practically important problem which has not been solved up to now. We give a two-sided estimate for the dielectric loss tangent of a composite and a solution of the problem for a composite in which the dielectric constant of the matrix is higher than that of the inclusion. The imaginary part  $\varepsilon''_{\text{mix}}$  of the averaged dielectric constant of a linear composite material can be calculated by the formula

$$\varepsilon''_{\text{mix}} = \frac{1}{|Y|} \int_Y \varepsilon''(\mathbf{x}) |\nabla N(\mathbf{x})|^2 d\mathbf{x},$$

where  $\varepsilon''(\mathbf{x})$  is the imaginary part of the dielectric constant of the components of the composite. In the case of small fields, this formula is also true for nonlinear composites [1].

The last equality can be expressed in terms of the dielectric loss tangent  $\tan \delta = \varepsilon''/\varepsilon'$  as

$$\varepsilon''_{\text{mix}} = \frac{1}{|Y|} \int_Y \tan \delta(\mathbf{x}) \varepsilon'(\mathbf{x}) |\nabla N(\mathbf{x})|^2 d\mathbf{x},$$

where  $\varepsilon'$  is the real part of the dielectric constant.

Introducing the dielectric loss tangent of the composite  $\tan \delta_{\text{mix}} = \varepsilon''_{\text{mix}}/A$  and using formula (9), we obtain

$$\tan \delta_{\text{mix}} = \int_Y \tan \delta(\mathbf{x}) \varepsilon'(\mathbf{x}) |\nabla N(\mathbf{x})|^2 d\mathbf{x} / \int_Y \varepsilon'(\mathbf{x}) |\nabla N(\mathbf{x})|^2 d\mathbf{x}. \quad (14)$$

In (14), the integrands contain the same term  $\varepsilon'(\mathbf{x}) |\nabla N(\mathbf{x})|^2$ . In view of this, from (14) we obtain the following estimate for the averaged dielectric loss tangent of the composite:

$$\min_{\mathbf{x} \in Y} \tan \delta(\mathbf{x}) \leq \tan \delta_{\text{mix}} \leq \max_{\mathbf{x} \in Y} \tan \delta(\mathbf{x}). \quad (15)$$

For the two-component composite material, the estimate becomes

$$\tan \delta_1 \leq \tan \delta_{\text{mix}} \leq \tan \delta_2,$$

where  $\tan \delta_1 \leq \tan \delta_2$  are the dielectric loss tangents of the constituent materials of the composite. According to this estimate, the average dielectric loss tangent of the composite material is between the largest and smallest dielectric loss tangents of the components of the composite, which agrees with the results of numerical calculations.

Let us consider a composite material with a ferroelectric matrix and dielectric inclusions. A composite of the ferroelectric–dielectric type is a high-contrast two-phase composite. From the results of numerical calculations [11], it follows that, in a high-contrast composite, the local energy  $\varepsilon'(\mathbf{x})|\nabla N(\mathbf{x})|^2$  (which determines both the averaged losses and the averaged dielectric constant of the composite) is squeezed out of the dielectric inclusions into the ferroelectric matrix. Therefore, Eq. (14) becomes

$$\tan \delta_{\text{mix}} = \tan \delta_M \int_M \varepsilon'(\mathbf{x})|\nabla N(\mathbf{x})|^2 d\mathbf{x} / \int_M \varepsilon'(\mathbf{x})|\nabla N(\mathbf{x})|^2 d\mathbf{x} = \tan \delta_M, \quad (16)$$

where  $\tan \delta_M$  is the dielectric loss tangent of the matrix.

According to formula (16), the averaged dielectric loss tangent of a composite material of the ferroelectric–dielectric type is equal to the dielectric loss tangent of the ferroelectric component. This formula explains the reason of the steadily high dielectric loss tangent of composites of the ferroelectric–dielectric type. This does not mean that designing composites with lower losses is impossible. On the contrary, from estimate (15), it follows that a reduction in the losses is also possible. However, the problem of loss reduction cannot be solved using simple technologies such as mixing–sintering ones.

**6. Conclusions.** The numerical solutions given above show that the control gain coefficient of composites can have values  $K > 1$  and reach values in the range  $K = 5\text{--}20$  (500–2000%) for feasible designs. Feasibility in this case means that the designs proposed here are not more complicated than the existing designs of modern composite materials [12, 13]. The relative complexity of the above designs for the materials considered here (a nonlinear ferroelectric with a large dielectric constant as the matrix and a linear dielectric with a small dielectric constant as inclusions) is due to the sensitivity of the design problem with respect to many available materials [14].

The dielectric loss tangent of the composite (for linear composites and for nonlinear composites in the case of weak fields) is between the maximum and minimum dielectric loss tangents of the composite components. This is evidence that the problem of reducing losses by using composites is well-posed. At the same time, for high-contrast composites of the ferroelectric–dielectric type, the loss tangent of the composite is close to the loss tangent of the ferroelectric.

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